The model

We consider $N$ bosons (or modes) $b_i$ $(n = 1,...,N)$ described by the annihilation (creation) operators $b_i$, $b_i^\dagger$ and an additional mode labeled $a$, which we call the root. The bosons interact through the resonant couplings of the root to each $b_i$. The satellite elements $b_i$ do not mutually interact. In the interaction picture, we consider the Hamiltonian

$$H_I = \sum_{i=1}^{N} g_i b_i b_i^\dagger + h.c. \quad (h = 1)$$

with $G_I$ the real and time-independent couplings. For $N = 1$, the propagator $\hat{U}(\tau) = \exp(-iH\tau)$ is similar to a beamsplitter superimposing mode $a$ to $b_1$. For $N > 1$, using the Lie algebra, we find that $\hat{U}(\tau)$ can be decomposed as

$$\hat{U}(\tau) = \left[ e^{i \sum_{j=1}^{N} \omega j b_j b_j^\dagger} \hat{B}_{\text{mod}}(\varphi,\tau) \right]$$

Here, $\omega_j = \sum_{k=1}^{N} \omega_k (1 \leq k \leq N)$ with $\omega_a = G_a = \tau$ and $\hat{B}_{\text{mod}}(\varphi,\tau)$ is a $\varphi$ phase-shift on mode $b_1$. $\hat{B}_{\text{mod}}(\varphi,\tau)$ is a beamsplitter operator with $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \varphi$.

The interaction configuration. Here, the satellite elements are embodied by $N$ independent bosons. The edges represent an interaction with the root.

Effective interferometric settings

The formal decomposition of $\hat{U}(\tau)$ can be immediately understood in terms of the following effective interferometer setting.

Many configurations can be engineered by simply reordering the coupling rates. For $g_a = g_b$ the model is a $1\times1$-coupling which allows for $1 \rightarrow N$ phase-conserving photonic splitters. If we prepare the root in $(|0\rangle + \sum a|0\rangle)_{0,a}$ and $\sum_{i=1}^{N} |i\rangle_{b_i}$, an $N+1$-mode GHZ state is created by this model.

Even more interestingly, perfect quantum state transfer between two satellite modes can be performed, for $N = 2$. Indeed, the equivalent set-up becomes a limited Mach-Zehnder interferometer.

$$\hat{U}(\tau)|\Phi(\tau)\rangle = |\Phi(\tau)\rangle - e^{-i\varphi} |\Phi(\tau)\rangle = |\Phi(\tau)\rangle + e^{-i\varphi} |\Phi(\tau)\rangle$$

At $\varphi = \sqrt{2\varphi}$, $\Phi = \tau = 1/|B_{\text{mod}}(\varphi,\tau)| = |B_{\text{mod}}(\varphi,\tau)|$.

Single excitation case

Consider the root in an initial state $|1\rangle$, the $b_j$'s being in $|g_{b_j}| = \{g_{b_j}\}$ (an excitation-number state). The evolution of this initial state is captured by considering a fictitious collective mode of its annihilation operator $i = \sum_{j=1}^{N} g_{b_j} b_{j}$. Thus, $\hat{U}(\tau) = \exp[-iG_{\text{ab}}^2 \hat{a}^\dagger \hat{a} + h.c.]$ and we have $|10\rangle_{b_1 b_2} \rightarrow |\cos \theta \rangle_{b_1} |\sin \theta \rangle_{b_2}$ where $\theta = \sqrt{a^\dagger a}$.

This entangled state of $N + 1$ modes can be pictorially described by permutation-invariant entanglement graphs corresponding to physical states of the network.

Continuous variable case

Our linear interaction maps a Gaussian state to a Gaussian state. For states in this class, necessary and sufficient conditions for the entanglement are well-known. This consider only a Gaussian state here. An $N$-partite Gaussian state is fully characterized by the knowledge of its $2N \times 2N$ variance matrix $V_{\Phi} = \{x_j, y_j\}$ ($j = 1, ..., N$). Here, $\Phi = \{g_1, g_2, ..., g_N\}$ is the vector of the quadratures. To characterize the state of our $N+1$ bosons, we need the variance matrix of their joint state after $\hat{U}(\tau)$. In phase space, the action of $\hat{U}(\tau)$ on $\chi$ is such that $\chi \rightarrow T^{-T} \chi T$ is the new variance matrix. Here, $T$ is the unitary matrix.

Possible setups

- Individual ensembles of neutral atoms simultaneously interacting with a photonic bus
- Bidimensional arrays of electrically coupled nano-electromechanical oscillators

References