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Test of quantum nonlocality for cavity fields

M. S. Kim[1,2] and Jinhyoung Lee[1,*]
[1]Department of Physics, Sogang University, CPO Box 1142, Seoul 100-611, Korea
[2]Department of Applied Mathematics and Theoretical Physics, The Queen's University of Belfast, Belfast BT7 1NN, United Kingdom
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There have been studies on the formation of quantum-nonlocal states in two spatially separate cavities. We suggest a nonlocal test for a field prepared in two cavities. We couple classical driving fields with cavities where a nonlocal state is prepared. Two independent two-level atoms are then sent through respective cavities to interact off-resonantly with the cavity fields. The atomic states are measured after the interaction. Bell's inequality can be tested by the joint probabilities of the two-level atoms being in their excited or ground states. We find that a violation of Bell's inequality can also be tested using a single atom sequentially interacting with the two cavities. Potential experimental errors are also considered. We show that with the present experimental condition of 5% error in the atomic velocity distribution, the violation of Bell's inequality can be measured.

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I. INTRODUCTION

Entangled states have been at the focus of discussions in quantum information theory encompassing quantum teleportation, computing, and cryptography. Two-body entanglement [1] allows diverse measurement schemes which can admit tests of quantum nonlocality [2,3]. Using the atom-field interaction in a high-Q cavity we can produce quantum entanglement between cavity fields, between atoms, and between a cavity field and an atom. An entangled pair of atoms have been experimentally generated using the cavity QED [4]. The entanglement of atoms and fields in the cavity can be utilized toward a realization of the controlled-NOT gate for quantum computation [5,6].

A pair of atoms can be prepared in an entangled state using the atom-field interaction in a high-Q cavity. The interaction of a single two-level atom with a cavity field brings about entanglement of the atom and the cavity field [7]. If the atom does not decay into other internal states after it comes out from the cavity, the entanglement will survive for long, and it can be transferred to a second atom interacting with the cavity field. The violation of Bell's inequality can be tested by a joint measurement of the atomic states.

There are proposals to entangle fields in two spatially separated cavities using atom-field interaction [8,9]. A two-level atom in its excited state passes sequentially through two resonant single-mode vacuum cavities, and is found to be in its ground state after the second-cavity interaction. If the interaction with the first cavity is equivalent to a $\pi/2$ vacuum pulse and the second-cavity interaction to a π pulse [6], then the atom could have deposited a photon either in the first cavity or in the second, so that the final state $|\Psi_f\rangle$ of the two-cavity field is [8]

$$|\Psi_f\rangle=\frac{1}{\sqrt{2}}(|1,0\rangle+e^{i\varphi}|0,1\rangle), \qquad (1)$$

where $|1,0\rangle$ denotes one photon in the first cavity and none in the second, and $|0,1\rangle$ the reverse. Using the entangled cavity field (1), an unknown atomic quantum state can also be teleported [6].

A three-level atom can act as a quantum switch to couple a vacuum cavity with an external classical coherent field. If the atom is in an intermediate state before it enters the cavity, the ac Stark shift between the intermediate and uppermost states brings about a resonant coupling of the cavity with the external field, so that the external field can be fed into the cavity. If the atom is initially in its lowermost state, the atom is unable to switch on the coupling between the cavity and the external field. Using the Ramsey interference and atomic quantum switch, Davidovich *et al.* suggested a coherent state entanglement $|\Psi_c\rangle$ between two separate cavities [9], $|\Psi_c\rangle=B_1|\alpha,0\rangle+B_2|0,\alpha\rangle$, where $|\alpha,0\rangle$ denotes the first cavity in the coherent state of the amplitude α and the second in the vacuum.

In this paper, we are interested in a test of nonlocality for an entangled field prepared in spatially separated cavities. Despite the suggestions on the production of entangled cavity fields, a test of the quantum entanglement, i.e., a measurement of the violation of Bell's inequality for the entangled cavity fields, has not been made. To test the quantum nonlocality, we first couple classical driving fields with cavities where a nonlocal state is prepared. Two independent two-level atoms are then sent through respective cavities to interact off-resonantly with the cavity fields. The atomic states are measured after the interaction. Bell's inequality can be tested by the joint probabilities of two-level atoms being in their excited or ground states. We find that quantum nonlocality can also be tested using a single atom sequentially interacting with the two cavities. We also consider potential experimental errors. The atoms normally have Gaussian velocity distribution with a normalized standard deviation less than 5%. We show that, even with the experimental errors caused by the velocity distribution, the test can be feasible.

II. BELL'S INEQUALITY BY PARITY MEASUREMENT

It is important to choose the type of measurement variables when testing nonlocality for a given state. Banaszek

*Electronic address: hyoung@quanta.sogang.ac.kr

and Wódkiewicz [10] considered even and odd parities of the field state as the measurement variables, where a state is defined to be in even (odd) parity if the state has even (odd) numbers of photons. The even- and odd-parity operators $\hat{O}_E$ and $\hat{O}_O$ are the projection operators to measure probabilities of the field having even and odd numbers of photons, respectively:

$$\hat{O}_E=\sum_{n=0}^{\infty}|2n\rangle\langle 2n|,\quad \hat{O}_O=\sum_{n=0}^{\infty}|2n+1\rangle\langle 2n+1|. \tag{2}$$

To test the quantum nonlocality for the field state of modes a and b, we define the quantum correlation operator based on the joint parity measurements,

$$\hat{\Pi}^{ab}(\alpha,\beta)=\hat{\Pi}^a_E(\alpha)\hat{\Pi}^b_E(\beta)-\hat{\Pi}^a_E(\alpha)\hat{\Pi}^b_O(\beta)-\hat{\Pi}^a_O(\alpha)\hat{\Pi}^b_E(\beta)+\hat{\Pi}^a_O(\alpha)\hat{\Pi}^b_O(\beta), \tag{3}$$

where the superscripts a and b denote the field modes and the displaced parity operator $\hat{\Pi}_{E,O}(\alpha)$ is defined as

$$\hat{\Pi}_{E,O}(\alpha)=\hat{D}(\alpha)\hat{O}_{E,O}\hat{D}^\dagger(\alpha). \tag{4}$$

The displacement operator $\hat{D}(\alpha)$ displaces a state by α in phase space. The displaced parity operator acts like a rotated spin projection operator in the spin measurement [2]. We can easily derive that the local hidden variable theory imposes the Bell's inequality [10]

$$|B(\alpha,\beta)|\equiv|\langle\hat{\Pi}^{ab}(0,0)+\hat{\Pi}^{ab}(0,\beta)+\hat{\Pi}^{ab}(\alpha,0)-\hat{\Pi}^{ab}(\alpha,\beta)\rangle|\le 2, \tag{5}$$

where we call $B(\alpha,\beta)$ the Bell function.

III. PARITY MEASUREMENT IN CAVITY QED

Englert *et al.* [11] proposed an experiment to determine the parity of the field in a high-**Q** single-mode cavity. Let us consider a far-off-resonant interaction of a two-level atom with a single-mode cavity field. If the detuning $\Delta=\omega_o-\omega$ of the atomic transition frequency ω_o from the cavity-field frequency ω is much larger than the Rabi frequency Ω, there is no energy exchange between the atom and the field, but the relative phase of the atomic states changes due to the ac stark shift [12]. The change of the phase depends on the number of photons in the cavity and on the state of the atom:

$$\begin{aligned}|e,\psi_f\rangle&\rightarrow\exp[-i\Theta(\hat{n}+1)]|e,\psi_f\rangle,\\ |g,\psi_f\rangle&\rightarrow\exp[i\Theta(\hat{n})]|g,\psi_f\rangle,\end{aligned} \tag{6}$$

where the atom-field state $|e,\psi_f\rangle$ denotes the atom in its excited state and the cavity field in $|\psi_f\rangle$. The phase $\Theta(\hat{n})$ is a function of the number of photons in the cavity and the atom-field coupling time and strength.

If a $\pi/2$ pulse is applied on a two-level atom before it enters the cavity, the atom, initially in its excited state, transits to a superposition state without changing the cavity field

$$|e,\psi_f\rangle\rightarrow\frac{1}{\sqrt{2}}[|e,\psi_f\rangle+ie^{i\phi_1}|g,\psi_f\rangle], \tag{7}$$

where ϕ_1 is determined by the phase of the pulse field. The atom, then, passes through a cavity and undergoes an off-resonant interaction with the cavity field where the atom-field coupling function is selected to be

$$\Theta(\hat{n})=\frac{\pi}{2}\hat{n}. \tag{8}$$

After the atom comes out from the cavity, the atom is allowed to interact with the second $\pi/2$ pulse. If the phases of the first and the second pulses are chosen to satisfy the relation

$$ie^{i(\phi_1-\phi_2)}=1, \tag{9}$$

the atom-field state after the second $\pi/2$ pulse is

$$\frac{1}{2}\{i[1-(-1)^{\hat{n}}]|e,\psi_f\rangle+e^{i\phi_2}[1+(-1)^{\hat{n}}]|g,\psi_f\rangle\}. \tag{10}$$

If the atom is detected in its excited state, the field has even parity. If the atom is detected in its ground state, the field has odd parity. The atom tells us the parity of the field [11].

IV. QUANTUM NONLOCALITY OF CAVITY FIELDS

To test the quantum nonlocality of the field, $|\psi_f^{ab}\rangle$, prepared in spatially separated two single-mode cavities, we use two two-level atoms as shown in Fig. 1(a). In this paper we assume that the mode structures of the cavities are identical and the atoms are independent identical atoms. The atoms are labeled a and b to interact, respectively, with the fields in the cavities a and b. Each atom is initially prepared in its excited state and sequentially passes through interaction zones of the first $\pi/2$ pulse, the cavity field, and the second $\pi/2$ pulse. The atom-field state then becomes

$$|e_a,e_b\rangle|\psi_f^{ab}\rangle\rightarrow|\varphi(\hat{n}_a,\hat{n}_b)\rangle|\psi_f^{ab}\rangle, \tag{11}$$

where $|\varphi(\hat{n}_a,\hat{n}_b)\rangle$ is the atomic state with the weights of the field operators ($\hat{n}_a$ and $\hat{n}_b$ are number operators of fields in the cavities a and b),

$$\begin{aligned}|\varphi(\hat{n}_a,\hat{n}_b)\rangle=&a(\hat{n}_a)a(\hat{n}_b)|e_a,e_b\rangle+a(\hat{n}_a)b(\hat{n}_b)|e_a,g_b\rangle\\&+b(\hat{n}_a)a(\hat{n}_b)|g_a,e_b\rangle+b(\hat{n}_a)b(\hat{n}_b)|g_a,g_b\rangle,\end{aligned} \tag{12}$$

where $a(\hat{n})=[e^{-i\Theta(\hat{n}+1)}-e^{i(\phi_1-\phi_2)}e^{i\Theta(\hat{n})}]/2$ and $b(\hat{n})=ie^{i\phi_2}[e^{-i\Theta(\hat{n}+1)}+e^{i(\phi_1-\phi_2)}e^{i\Theta(\hat{n})}]/2$. Choosing appropriate conditions for the atom-field couplings and pulse phases as shown in Eqs. (8) and (9), the atom-field state becomes

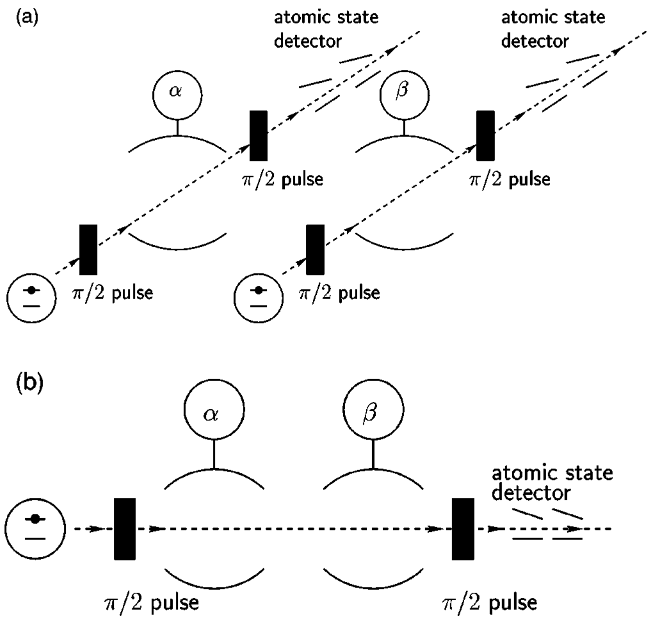

FIG. 1. Schematic diagram of the quantum nonlocality test for cavity fields. (a) Two two-level atoms pass through two cavities, and joint measurements of atomic levels are performed after the cavity interaction. (b) A single two-level atom passes sequentially through two cavities, and a measurement of atomic level is performed after the atom-field interaction in the cavities.

$$\frac{1}{4}\{i[1-(-1)^{\hat{n}_a}]|e_a\rangle+e^{i\phi_2}[1+(-1)^{\hat{n}_a}]|g_a\rangle\} \times\{i[1-(-1)^{\hat{n}_b}]|e_b\rangle+\mathrm{e}^{i\phi_2}[1+(-1)^{\hat{n}_b}]|g_b\rangle\}|\psi_f^{ab}\rangle. \qquad (13)$$

If the atoms are jointly found in their excited states, then we know that both cavities are in odd-parity states. The joint probability P_{ee} of the atoms being in their excited states is related to the expectation value of the following joint parity operator:

$$P_{ee}=\langle\hat{\Pi}_O^a(0)\hat{\Pi}_O^b(0)\rangle. \qquad (14)$$

In Eqs. (3) and (5), it is seen that we need to know the joint parities of the *displaced* original fields to test the quantum nonlocality. To displace the cavity field, external stable fields are coupled to the cavities as shown in Fig. 1(a) [13]. After a nonlocal field state is prepared in the cavities, we couple the cavities with the external fields to displace the original nonlocal field, then send two independent atoms through the respective cavities. The $\pi/2$ pulses shine atoms before and after the cavity interaction to provide Ramsey interference effects. The atomic states are detected after the second $\pi/2$ pulses. $P_{ee}(\alpha,\beta)$ denotes the joint probability of atoms being in their excited states when the original fields in the cavities a and b are displaced by α and β, respectively. The expectation value of the quantum correlation operator in Eq. (3) is obtained by the joint probabilities

$$\langle\hat{\Pi}^{ab}(\alpha,\beta)\rangle=P_{ee}(\alpha,\beta)-P_{eg}(\alpha,\beta) -P_{ge}(\alpha,\beta)+P_{gg}(\alpha,\beta). \qquad (15)$$

If there are any displacement factors α and β which result in a violation of Bell's inequality in Eq. (5), the field originally prepared in the cavities is quantum mechanically nonlocal.

After a closer look at Eq. (3), we find that we do not need the individual parity of each cavity field to test inequality (5). We only need the parity of the total field. Instead of sending two atoms to cavities, we now send a single two-level atom sequentially through cavities, as shown in Fig. 1(b). The atom is initially prepared in its excited state and undergoes $\pi/2$ pulse interactions before and after the cavity interaction. The atom-field coupling strength is selected to satisfy Eq. (8), and the phases ϕ_a and ϕ_b of the two $\pi/2$ pulses are chosen as $\exp[i(\phi_a-\phi_b)]=1$; then the atom-field state becomes

$$|e,\psi_f^{ab}\rangle\rightarrow-\frac{1}{2}[1+(-1)^{\hat{n}_a+\hat{n}_b}]|e,\psi_f^{ab}\rangle +\frac{i}{2}[1-(-1)^{\hat{n}_a+\hat{n}_b}]|g,\psi_f^{ab}\rangle. \qquad (16)$$

The external stable fields are taken to be coupled with the cavities to displace the cavity fields. The probability $P_e(\alpha,\beta)$ of the atom being in its excited state, after having passed displaced cavity fields and $\pi/2$ pulses, is the expectation value of the parity operators,

$$P_e(\alpha,\beta)=\langle\hat{\Pi}_O^a(\alpha)\hat{\Pi}_O^b(\beta)+\hat{\Pi}_E^a(\alpha)\hat{\Pi}_E^b(\beta)\rangle, \qquad (17)$$

where α, β denote the displacements of the fields in cavities a and b. Similarly, the probability of the atom being in its ground state $P_g(\alpha,\beta)$ is found to be related to the odd parity of the total fields. The expectation value of the quantum correlation function operator in Eq. (3) is simply

$$\langle\hat{\Pi}^{ab}(\alpha,\beta)\rangle=P_e(\alpha,\beta)-P_g(\alpha,\beta). \qquad (18)$$

This does not tell us the parity of each mode but the parity of the total field, which is enough to test the violation of Bell's inequality.

V. REMARKS

We have suggested a simple way to test the quantum nonlocality of cavity fields by measuring the states of atoms after their interaction with cavity fields. The test does not require a numerical process on the measured data. The difference in the probability of a single two-level atom being in its excited and ground states is directly related to the test of quantum nonlocality. In fact, this can also be used to reconstruct the two-mode Wigner function, as the mean parity of the field is proportional to the two-mode Wigner function [10,14,15]:

$$W(\alpha,\beta)=(2/\pi)^2\langle\hat{\Pi}^{ab}(\alpha,\beta)\rangle. \qquad (19)$$

Experimental error can easily occur due to fluctuation in the atom-field coupling strength and time. The atom-field coupling function $\Theta(\hat{n})$ depends on the mode structure of the cavity field and on the time it takes for the atom to interact with the cavity field [11]. Because the atomic velocity has some fluctuations, the interaction time is subject to experimental error [16]. Another error source may be the $\pi/2$ pulse operation. We now analyze the possibility to measure the violation of quantum nonlocality in the potential experimental situation.

The test of quantum nonlocality using the two-atom scheme in Fig. 1(a) is considered with potential experimental errors. The error in the atom-field coupling function is denoted by $\Delta\Theta(\hat{n})$, which is the departure of the experimental value $\Theta(\hat{n})$ from the required value $\Theta_0(\hat{n})$,

$$\Delta\Theta(\hat{n})=\Theta(\hat{n})-\Theta_0(\hat{n})=\delta\hat{n}, \tag{20}$$

where $\Theta_0(\hat{n})=(\pi/2)\hat{n}$. The relative phases of atomic states given by the $\pi/2$-pulse interactions are also subject to experimental errors. We take the phase error $\Delta\phi$ as

$$\Delta\phi=(\phi_2-\phi_1)-\phi_0, \quad i\exp(i\phi_0)=1. \tag{21}$$

Note that the atomic state measurement is equivalent to the parity measurement as in Eq. (14) only when $\Theta(\hat{n})=\Theta_0(\hat{n})$ and $\phi_2-\phi_1=\phi_0$.

The errors in the atom-field coupling and phases of the $\pi/2$ pulses bring about the departure $\Delta\Pi^{ab}(\alpha,\beta)$ of the joint atomic state probabilities from the expectation value of the parity operators in Eq. (15). The mean error of $\Delta\Pi^{ab}(\alpha,\beta)$ is calculated up to the second order of δ and $\Delta\phi$ as follows:

$$\begin{aligned}\Delta\Pi^{ab}(\alpha,\beta)&\equiv P_{ee}(\alpha,\beta)-P_{eg}(\alpha,\beta)-P_{ge}(\alpha,\beta)\\&\quad+P_{gg}(\alpha,\beta)-\langle\hat{\Pi}^{ab}(\alpha,\beta)\rangle\\&=-2\langle\psi_f^{ab}|\hat{\Pi}^{ab}(\alpha,\beta)\{\Delta[\hat{n}_a(\alpha)]\\&\quad+\Delta[\hat{n}_b(\beta)]\}|\psi^{ab}\rangle,\end{aligned} \tag{22}$$

where

$$\begin{aligned}\Delta[\hat{n}_{a,b}(\alpha)]&=\frac{1}{4}[2\hat{n}_{a,b}(\alpha)+1]^2\delta^2\\&\quad+\frac{1}{2}[2\hat{n}_{a,b}(\alpha)+1]\delta\Delta\phi+\frac{1}{4}(\Delta\phi)^2,\end{aligned} \tag{23}$$

and $\hat{n}_{a,b}(\alpha)=\hat{D}^\dagger(\alpha)\hat{n}_{a,b}\hat{D}(\alpha)$ are the displaced number operators for the field modes in cavities a and b. The mean error $\Delta B(\alpha,\beta)$ of the Bell function measurement in Eq. (5) is given by

$$\begin{aligned}\Delta B(\alpha,\beta)&=\Delta\Pi^{ab}(0,0)+\Delta\Pi^{ab}(\alpha,0)\\&\quad+\Delta\Pi^{ab}(0,\beta)-\Delta\Pi^{ab}(\alpha,\beta).\end{aligned} \tag{24}$$

Consider an explicit example of a quantum nonlocal field (1) for an illustration of the experimental errors. For simplicity, we take the phase factor to be zero, i.e., $\varphi=0$. We know from earlier work [10] that Bell's inequality is maximally violated with $B\sim-2.19$ when $\alpha=-\beta$ and $|\alpha|^2\sim0.1$. Substituting $|\Psi_f\rangle$ of Eq. (1) into $|\psi_f^{ab}\rangle$ of Eq. (22), $\Delta\Pi^{ab}(\alpha,\beta)$ is

$$\begin{aligned}\Delta\Pi^{ab}(\alpha,\beta)&\approx-2\langle\psi_f^{ab}|\hat{\Pi}^{ab}(\alpha,\beta)|\psi_f^{ab}\rangle\\&\quad\times\langle\psi_f^{ab}|\{\Delta[\hat{n}_a(\alpha)]+\Delta[\hat{n}_b(\beta)]\}|\psi_f^{ab}\rangle\\&=-2\langle\hat{\Pi}^{ab}(\alpha,\beta)\rangle\{c_1(\alpha,\beta)\delta^2\\&\quad+c_2(\alpha,\beta)\delta\Delta\phi+(\Delta\phi)^2\},\end{aligned} \tag{25}$$

where the mean-field approximation has been used [17]. The expectation value $\langle\hat{\Pi}^{ab}(\alpha,\beta)\rangle=(2|\alpha-\beta|^2-1)e^{-2(|\alpha|^2+|\beta|^2)}$, and the parameters $c_1(\alpha,\beta)=2(|\alpha|^4+|\beta|^4)+\frac{13}{2}(|\alpha|^2+|\beta|^2)+5$ and $c_2(\alpha,\beta)=2(|\alpha|^2+|\beta|^2)+4$. The mean error ΔB for $\alpha=-\beta\approx\sqrt{0.1}$ is then given by

$$\Delta B\sim10.2\delta^2+8.1\delta\Delta\phi+2.0(\Delta\phi)^2. \tag{26}$$

The probing atoms normally have a Gaussian velocity distribution which causes the errors δ and $\Delta\phi$. When we consider the ensemble average over atoms, the second term vanishes in Eq. (26), and the first and third terms finally contribute to degrade the value of the Bell function.

For the test of nonlocality using single atoms as shown in Fig. 1(b), the mean error ΔB is similarly obtained as

$$\Delta B\sim16.3\delta^2+8.2\delta\Delta\phi+1.0(\Delta\phi)^2. \tag{27}$$

This is slightly larger than error (26) for the two-atom scheme. The error enhancement in the single-atom scheme is due to the fact that the experimental errors are multiplied as the atom passes through two cavities. For the two-atom scheme, the error is the sum of errors that occur in each atom interaction with the cavity field and $\pi/2$ pulses. We find that when the standard deviation of the atomic velocity distribution is 5%, $\Delta B\sim0.06$ for the two-atom scheme and $\Delta B\sim0.10$ for the single-atom scheme, which still allows the observation of the violation of Bell's inequality.

If the Q factor of a cavity is high, the cavity is very much closed. When an atom passes through the cavity walls the atom can lose information about the phases of atomic states, so that the scheme suggested in this paper cannot be used. However, recently, Nogues *et al.* suggested a way to implement $\pi/2$ pulses and cavity field interactions inside the cavity [18]. If this scheme is applied there will not be the problem of losing Q value to retain atomic phase information.

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