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On the Disentangling Process for Two-Mode Squeezed State

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A pure state decoheres into a mixed state as the system entangles with an environment which is initially in a pure state. However, it is not definite that the system becomes entangled with a confined environment with which it only ever interacts. We investigate the disentangling mechanism by considering the quantum correlation between a two-mode squeezed state and a thermal environment.

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I. INTRODUCTION

Decoherence has been studied in the context of quantum-classical correspondence, providing a quantum-to-classical transition of a system [1]. The effects of the decoherence have been intensively studied in quantum information theory [2–4]. A pure single-mode field becomes a mixed state and loses its quantum nature by decoherence in an environment. It is widely accepted that the decoherence process may be understood as a process of entangling the system and its environment, which is composed of a large (normally infinite) number of independent modes [5,6]. The change of the system entropy may be regarded as an indication of the system-environment entanglement [5].

Most of the studies on decoherence have focused on a single-body system [7], because the decoherence process of a many-body system is expected to be a straightforward extension of a single-body system in a Markov environment, if it is separable. However, if it is entangled, the decoherence mechanism may impose a different nature, which, in addition, is relevant to many intriguing arguments in quantum mechanics, including Zurek's postulate [1] on our observation of a classical state. Schrödinger's cat, for instance, is observed in either the alive or the dead state, and not in their superposition. In the measurement process, a system $\mathcal{S}$ is fully entangled with the measurement apparatus $\mathcal{M}$, such that $\mathcal{S}+\mathcal{M}$ is in the state

$$a|\text{alive}\rangle|\mathcal{M}_a\rangle + b|\text{dead}\rangle|\mathcal{M}_d\rangle \quad (1)$$

where $|\mathcal{M}_a\rangle$ and $|\mathcal{M}_d\rangle$ are orthogonal. The apparatus and the environment $\mathcal{E}$ are further entangled due to their interaction. It was argued that the chain interaction gives rise to GHZ entanglement for the composition of $\mathcal{S}+\mathcal{M}+\mathcal{E}$ [1]. Upon elimination of the environment, the system and the apparatus enter a classically correlated state, so that the gauge of the apparatus indicates the classical state of the system: Schrödinger's cat is either alive or dead. In these approaches, the environment is commonly assumed to be in a pure state. However, if it is confined to the variables that interact with a system ($\mathcal{S}+\mathcal{M}$ in the example), what kind of entanglement is involved in our observation of classical states? In this paper, we investigate the disentangling process by studying the quantum correlation of a two-mode entangled continuous-variable system and an environment in thermal equilibrium.

A continuous-variable state is defined in an infinite-dimensional Hilbert space, and it is convenient to work with its Wigner function [8], $W(\tilde{\mathbf{x}})$, in phase space. For an N-mode field, the phase space is coordinated by quadrature variables, $\tilde{\mathbf{x}} = \{q_1, p_1, \cdots, q_N, p_N\}$. Throughout the paper, a vector is denoted in bold face and an operator by a hat. A light field of a continuous-variable system propagating through a fiber or a free space is normally considered as a thermal environment. The dynamics of the field coupled to the thermal environment is governed by the Fokker-Planck equation in the interaction picture [8],

$$\frac{\partial W(\tilde{\mathbf{x}})}{\partial \tau} = \frac{\gamma}{2}\sum_{i=1}^{2N}\left[\frac{\partial}{\partial \tilde{x}_i}\tilde{x}_i + \frac{\tilde{n}}{4}\frac{\partial^2}{\partial \tilde{x}_i^2}\right]W(\tilde{\mathbf{x}}), \quad (2)$$

where γ is the energy decay rate of the system and

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$\tilde{n} = 2\bar{n}+1$, with $\bar{n} = [\exp(\hbar\omega/k_BT)-1]^{-1}$, the average number of thermal photons at temperature T. k_B is the Boltzmann constant. The first term in the bracket describes an energy dissipation, whilst the second is due to diffusion which causes the decoherence of the system.

By using the Fokker-Planck equation (2), it is difficult to directly investigate the quantum correlation between the system and the environment as it is derived by tracing over all environmental variables. In this paper, we show that the system interacting with the infinite environmental variables is equivalent to a system interacting with a small number of *effective environmental variables* such that their interactions result in the identical Fokker-Planck equation of the system. For instance, a single-mode field in a thermal environment is equivalent to one interacting with a single mode "environment" in a thermal state. The sufficiency of the few environmental variables in describing the decoherence enables one to manipulate an experiment so as to investigate the decoherence by introducing such effective environmental variables and even controlling their couplings to the system. By keeping the environmental variable(s) one can study the entanglement nature of the system and its environment.

II. EFFECTIVE ENVIRONMENT APPROACH

Reduced dynamics of an open quantum system has commonly been investigated by a master equation with reduced density operator or Fokker-Planck equation with quasi-probability function like the Wigner function [9]. By the master equation (or Fokker-Planck equation), however, it is difficult to investigate the quantum correlation between the system and the environment, as it is derived by tracing over all environmental variables [10]. We suggest another approach by introducing the notion of an "effective environment". It will be shown that the system interacting with the large number of environmental degrees of freedom (which we call "environmental variables") is equivalent to a system interacting with a small number of effective environmental variables in the sense that their interactions result in the same master (or Fokker-Planck) equation of the system.

The existence of the small number of effective environmental variables, that lead to the correct and equivalent description of the system's dynamics, is predicted by two observations. First, at a given time τ, the density operator $\hat{\rho}$ of the system S can always be purified to a pure state $|\psi\rangle$ of the larger composite system consisting of the system S and an ancillary system R, such that $\hat{\rho}$ is obtained by tracing the pure state over the ancillary system, *i.e.*, $\hat{\rho} = \mathrm{Tr}_r|\psi\rangle_{sr}\langle\psi|$. For the purification, an ancillary system is required to have its Hilbert space larger than or equal to that of the system [11]: For instance, a qubit may suffice for the ancillary system if the system is a qubit. As the composite system $S+R$ provides all physical descriptions relating to the system, the ancillary system is called effective variable(s) (certain collective degrees of freedom) of the environment at the given time τ. Secondly, the effective variables may be mobile over the environment as the interaction time τ passes. The mobility of the effective variables can be absorbed by time-dependent coupling constants, which, in turn, make the effective variables stationary. The set of time-dependent coupling constants and the effective variables contains all the information of the environment that governs the dynamics of the system. We call this set an effective environment.

We shall show that a single-mode system S and a single mode of an effective environment R leads to the Fokker-Planck equation (2) for the Markovian amplitude damping channel. The interaction Hamiltonian between the two modes S and R, in the interaction picture, is assumed to be

$$\hat{H}_I(\tau) = i\lambda(\tau)\left(\hat{s}\hat{r}^\dagger - \hat{s}^\dagger\hat{r}\right), \tag{3}$$

where $\hat{s}$ ($\hat{r}$) is the annihilation operator for the system S (effective environment R). The evolution operator $\hat{U}_I(\tau)$ for the interaction Hamiltonian $\hat{H}_I(\tau)$ in Eq. (3) is equivalent to a beam-splitter operator with the system mode S and the effective environment R as its input modes:

$$\hat{U}_I(\tau) = \exp[\Lambda(\tau)(\hat{s}\hat{r}^\dagger - \hat{s}^\dagger\hat{r})], \tag{4}$$

where $\Lambda(\tau) = \int_0^\tau \lambda(\tau')d\tau'$ gives the transmittivity and, $t^2(\tau) = \cos^2\Lambda(\tau)$. On keeping the transmitted energy finite, $t^2(\tau) = \exp(-\gamma\tau)$, the time-dependent beam-splitter operator $\hat{U}_I(\tau)$ leads to the Fokker-Planck equation (2) if the mode R is initially in a thermal state with average photon number $\bar{n}$.

In order to derive the Fokker-Planck equation, it is convenient to use the characteristic function for a single mode, defined as

$$\chi(\xi) = \mathrm{Tr}\left(\hat{T}(\xi)\hat{\rho}\right), \tag{5}$$

where $\hat{\rho}$ is a density operator of the single mode and $\hat{T}(\xi) = \exp(\xi\hat{s}^\dagger - \xi^*\hat{s})$ the displacement operator [8,12]. The Wigner function $W(\alpha)$ is given by Fourier transformation to the characteristic function $\chi(\xi)$ as

$$W(\alpha) = \frac{1}{\pi}\int d^2\xi \exp\left(\alpha\xi^* - \alpha^*\xi\right)\chi(\xi). \tag{6}$$

A beam splitter operation is described by a convolution law between two Wigner functions of input modes. The convolution implies that for each output mode its characteristic function results from a simple product of the characteristic functions of the two input modes [13]. As the evolution operator $\hat{U}_I(\tau)$ is a beam splitter operator, the characteristic function of the system S at a time τ is given by

$$\chi(\xi,\tau) = \chi_s(t(\tau)\xi)\chi_r(r(\tau)\xi), \tag{7}$$

where $\chi_s(\xi)$ and $\chi_r(\xi)$ are characteristic functions for the initial states of the system S and the environment R, respectively. Here, the transmittivity $t(\tau)$ is given below Eq. (4) and the reflectivity $r^2(\tau) = 1 - t^2(\tau)$. The dynamic equation of $\chi(\xi, \tau)$ is given by the time derivative in Eq. (7) as

$$\dot{\chi}(\xi,\tau) = \dot{\chi}_s(t(\tau)\xi)\chi_r(r(\tau)\xi) + \chi_s(t(\tau)\xi)\dot{\chi}_r(r(\tau)\xi). \quad (8)$$

Noting that the effective environment is initially in a thermal state, $\chi_r(r(\tau)\xi) = \exp(-\tilde{n}r^2(\tau)|\xi|^2/2)$. By performing the Fourier transformation of Eq. (8), one can easily obtain the Fokker-Planck equation (2).

From the present approach, the generalization to the N-mode system is straightforward, and further it can easily be extended to incorporate a general Gaussian, *i.e.*, squeezed thermal, environment. This was proven in an alternative approach using an infinite beam-splitter model [13]. We stress that the present approach is general, as the principal idea is applicable to a non-Markovian environment and even to a finite-dimensional system [14].

III. DISENTANGLING PROCESS

A two-mode squeezed state of modes a_1 and a_2, which can be generated by a nondegenerated optical parametric amplifier, is the most renowned and experimentally-relevant entangled state for continuous variables [15]. We assume that the mode a_2 of the system modes interacts with the environment, which is the circumstance of the measurement apparatus interacting with the environment. The two-mode squeezed state is a Gaussian state, which is represented by its characteristic function in the form of $\chi(\mathbf{x}) = \exp(-\mathbf{x}\mathbf{V}\mathbf{x}^T/4)$. $\mathbf{V}$ is the correlation matrix whose elements determine the mean quadrature values of the field: $V_{ij} = \langle(\hat{x}_i\hat{x}_j + \hat{x}_j\hat{x}_i)\rangle$. Here, we neglect linear terms in $\mathbf{x}$ because they do not affect entanglement properties. For a two-mode squeezed state, the correlation matrix $\mathbf{V}_s$ is

$$\mathbf{V}_s = \begin{pmatrix} \cosh(2s)\mathbb{1} & \sinh(2s)\boldsymbol{\sigma}_z \\ \sinh(2s)\boldsymbol{\sigma}_z & \cosh(2s)\mathbb{1} \end{pmatrix}, \quad (9)$$

where $\mathbb{1}$ is the 2×2 unit matrix, $\boldsymbol{\sigma}_z$ the Pauli matrix and s the squeezing parameter [16].

Consider the correlation matrix of the two system modes a_1 and a_2 and the effective environment b. The correlation matrix before the interaction is given by $\mathbf{V}_0 = \mathbf{V}_s \oplus \tilde{n}\mathbb{1}$. The evolution operator $\hat{U}_I(\tau)$ in Eq.(4) is now described by the matrix

$$\mathbf{U}_I = \begin{pmatrix} \mathbb{1} & 0 & 0 \\ 0 & t\mathbb{1} & -r\mathbb{1} \\ 0 & r\mathbb{1} & t\mathbb{1} \end{pmatrix}, \quad (10)$$

where $r^2 = 1 - t^2$ and the dependence on τ is omitted. Then, the correlation matrix for the total system after the interaction is given as $\mathbf{V}_c = \mathbf{U}_I\mathbf{V}_0\mathbf{U}_I^T$.

The entanglement structure is investigated with a separability condition that a two-mode Gaussian state is separable if and only if the partially momentum-reversed correlation matrix (or the partially transposed density operator) satisfies the uncertainty principle [17]. The condition was extended to a biseparability condition between a single mode and a group of N modes by Werner and Wolf [18], which states that a Gaussian field of $1 \times N$ modes is biseparable if and only if

$$\boldsymbol{\Lambda}\mathbf{V}\boldsymbol{\Lambda} - \frac{1}{2}\boldsymbol{\sigma}_y^{\oplus(N+1)} \geq 0, \quad (11)$$

where $\boldsymbol{\Lambda}$ is a partial momentum-reversal matrix on the particular single mode, $\mathbf{V}$ the correlation matrix of $1 \times N$ modes, and $\boldsymbol{\sigma}_y$ the Pauli matrix. Here, $\boldsymbol{\sigma}_y^{\oplus(N+1)} \equiv \boldsymbol{\sigma}_y \oplus \boldsymbol{\sigma}_y \oplus ... \oplus \boldsymbol{\sigma}_y$.

Let us consider pairwise entanglement between any two of the three modes. The correlation matrix $\mathbf{V}_c$ for the modes a_1 and a_2 is given as

$$\mathbf{V}_c(a_1, a_2) = \begin{pmatrix} \cosh(2s)\mathbb{1} & t\sinh(2s)\boldsymbol{\sigma}_z \\ t\sinh(2s)\boldsymbol{\sigma}_z & (t^2\cosh 2s + r^2\tilde{n})\mathbb{1} \end{pmatrix}. \quad (12)$$

Using Simon's criterion [17], the system modes a_1 and a_2 are separable when $t^2 \leq t^2_{a_1a_2} = \bar{n}/(1+\bar{n})$ and $s \neq 0$. For the entanglement of the system mode a_1 and the environment b, $\mathbf{V}_c(a_1, b)$ is equivalent to $\mathbf{V}_c(a_1, a_2)$ if r and t are interchanged. Thus, due to the symmetry the separability condition now reads as $t^2 \geq t^2_{a_1b} = 1 - t^2_{a_1a_2}$. In both cases the results are independent of the initial entanglement of the system, depending only on the temperature of the environment and the interaction time τ as $t^2 = \exp(-\gamma\tau)$. Modes a_2 and b are always separable as their quasiprobability P function always exists [19]. Tracing over a_1, the mode a_2 becomes a thermal state with the number of effective thermal photons $\bar{n}_s = (\cosh 2s - 1)/2$. A product of thermal states has a positive definite P function and the action of the beam splitter only transforms the coordinates of the input P function: $P_{a_2}(\tilde{x}_1)P_b(\tilde{x}_2) \xrightarrow{bs} P_{a_2}(t\tilde{x}_1 - r\tilde{x}_2)P_b(t\tilde{x}_2 + r\tilde{x}_1)$.

Figure 1 presents the entanglement structure among the system modes and the effective environment. The solid lines are the boundaries of entanglement of the system mode a_1 and the effective environment b, $t^2_{a_1b}$, and of the two system modes a_1 and a_2, $t^2_{a_1a_2}$. These lines are obtained from the separability condition described above, by using $\mathbf{V}_c$. For comparison, we consider $N = 100$ beam splitters modeling the interaction of the system with the environment consisting of N modes and calculate the biseparability of the 1×100-mode field by using Giedke *et al.*'s computational analysis [20]. The computational results of entanglement are denoted by circles and dots. We find that the two methods are exactly consistent.

The three modes a_1, a_2, and b compose a tripartite system. Many-body entanglement for continuous-variables

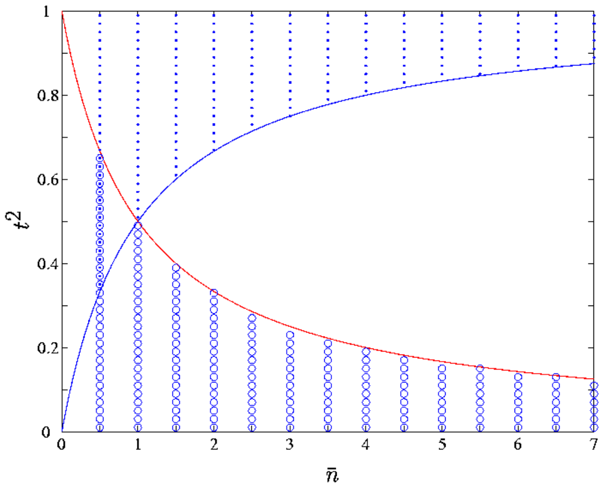

Fig. 1. Diagram of entanglement for a two-mode squeezed state interacting with a thermal environment of average photon number $\bar{n}$. $t^2 = \exp(-\gamma\tau)$ is the transmittivity of the beam splitter. The solid lines are the boundaries of entanglement of a_1 and b and of a_1 and a_2, which are obtained from the separability condition. The circles and dots are found by computational analysis with $N = 100$ beam splitters. The circles indicate that the system mode a_1 and the thermal environment are entangled, and the dots indicate that the system modes a_1 and a_2 are entangled.

has been studied extensively by using beam splitters and single-mode squeezed states [21]. Giedke *et al.* classified types of entanglement for a three-mode Gaussian field in terms of the biseparability [20]. A three-mode field is biseparable when any grouping of three modes into two is separable. When a three-mode field is not biseparable in any of the three possible ways, it is called fully entangled. A fully-entangled tripartite system may be further classified in terms of pair-wise entanglement; for qubits, two kinds have been discussed [22,23], one of which is Greenberg-Horne-Zeilinger (GHZ) entanglement [24] and the other is W-entanglement. A GHZ-entangled state becomes separable when any one particle is traced out, while a W-entangled state is pairwise entangled for any pair of the three particles. On the other hand, there is another kind of entanglement, two-way entanglement, for a fully-entangled tripartite d-dimensional system ($d \geq 3$) [25]. One example for three particles labeled as a, b and c is pairwise entanglement of a and b and of b and c but the particles a and c are separable in addition to their being fully entangled.

It is found both computationally, and analytically by using the biseparability condition in Eq. (11) that the tripartite system of a_1, a_2 and b is fully entangled if $0 < t^2 < 1$ and $s \neq 0$. This fact is independent of the temperature of the environment so that the tripartite system is fully entangled over the whole region in Fig. 1. The tripartite system is two-way entangled, *i.e.*, a_1-a_2 and a_1-b modes are entangled if $\bar{n} < 1$ and $t^2_{a_1a_2} < t^2 < t^2_{a_1b}$. There are two regions where one pair of the three-mode field is entangled. In the region of $\bar{n} > 1$ and $t^2_{a_1b} \leq t^2 \leq t^2_{a_1a_2}$, the three modes are GHZ-entangled. Here, we note that GHZ entanglement of an entangled system with an environment is an important source of its loss of the initial pair-wise entanglement. This is clearly seen for $\bar{n} > 1$ in Fig. 1. The two modes of the system are initially entangled, but as τ increases such that GHZ entanglement emerges, they lose their initial entanglement. Finally, the entanglement is transferred to between the system mode a_1 and the effective environmental mode. For $\bar{n} < 1$ the decoherence process is different with initially a_1-a_2 modes being entangled, then two-way entanglement and finally a_1-b being entangled.

There is an observable to distinguish the different routes. For a Gaussian state with correlation matrix $\mathbf{V}$, $\mathrm{Tr}\hat{\rho}^2 = 1/\sqrt{\det\mathbf{V}}$ [16], the mixedness of the Gaussian state may be measured by $M = \sqrt{\det\mathbf{V}} - 1$, which is 0 when the state is pure and grows as the system becomes mixed. This measure is relevant to experiment, as all the elements of the correlation matrix are measurable bu using homodyne detectors. We examine the mixedness of the system at the time when it loses its entanglement, *i.e.*, $t^2 = t^2_{a_1a_2}$:

$$M_e = 2\left[\left(2 - \frac{1}{\bar{n}+1}\right)\cosh^2 s - 1\right]. \tag{13}$$

We see that the mixedness M_e grows with $\bar{n}$ and reaches its half point M_1 when $\bar{n} = 1$. This result holds particularly for a two-mode squeezed state, but may indeed suggest that the mixedness of the system is a strong indicator of the mechanism of system-environment entanglement. Thus, for $M_e < M_1$, decoherence into two-way entanglement occurs, whilst for $M_e > M_1$ GHZ entanglement occurs.

One may go beyond Markov approximation and extend the present approach to incorporate the finite N modes $\{\hat{b}_j\}$ of the environment with natural frequencies ω_j, that influence the system with arbitrary coupling constants λ_j. The total Hamiltonian ($\hbar = 1$) is given as

$$\hat{H} = 2\sum_{i=1}^{2}\omega_0\hat{L}_{a_i} + 2\sum_{j=1}^{N}\omega_j\hat{L}_{b_j} + 2\sum_{j=1}^{N}\lambda_j\hat{M}_{a_2b_j}, \tag{14}$$

where the Hermitian operators $\hat{L}_u = (2\hat{u}^\dagger\hat{u}+1)/4$ and $\hat{M}_{uv} = (\hat{u}^\dagger\hat{v} + \hat{u}\hat{v}^\dagger)/2$ with $u, v = a_1, a_2$ and b_j. Here, the first (second) term is the free Hamiltonian of the two modes of system (the environmental modes) and the third term is the interaction Hamiltonian between the second mode of the system and the environmental modes. On introducing new Hermitian operators $N_{uv} = -i(\hat{u}^\dagger\hat{v} - \hat{u}\hat{v}^\dagger)/2$ and letting $\hat{L}^{\pm}_{uv} = \hat{L}_u \pm \hat{L}_v$, the set $A = \{L^{\pm}_{uv}, M_{uv}, N_{uv}\}$ forms a Lie algebra [26] with the commutation relations

$$[\hat{A}_i, \hat{A}_j] = C_{ijk}\hat{A}_k, \tag{15}$$

where $\hat{A}_i \in A$ and C_{ijk} is a structure constant. The evolution operator $\hat{U} = \exp(-i\hat{H}\tau)$ can now be represented by the elements (generators) of the Lie algebra A as

$$\hat{U} = \exp\left(i\sum_i g_i(\tau)\hat{A}_i\right) = \prod_i \exp\left(ig'_i(\tau)\hat{A}_i\right), \quad (16)$$

where $g_i(\tau)$ and $g'_i(\tau)$ are real coefficients depending on the time τ. Note that the evolution operator $\hat{U}$ consists of the rotators generated by $\hat{L}^{\pm}_{uv}$ and the beam splitters generated by $\hat{M}_{uv}$ or $\hat{N}_{uv}$. We know that the thermal reservoir has rotation symmetry so that the rotation should not change the nature of quantum correlation with the environment. Then, only the beam-splitter operators are left and the argument in this paper should hold in this very general case: We know that any combination of rotators and beamsplitters does not bring about entanglement in the output fields when the input fields are classical [27], so, even in this case, there is no entanglement between a_2 and environment modes. Further details on the entanglement nature within the non-Markovian environment interaction will be discussed elsewhere.

IV. CONCLUSIONS

A two-mode squeezed state [8], which is represented in the Fock basis as $\sum \phi_n|n\rangle|n\rangle$, $\phi_n = (\tanh s)^n/\cosh s$, may be understood as the coupling of the measurement apparatus to the system in the state $\sum \phi_n|n\rangle$ as in Eq.(1). Zurek attributed not being able to observe a quantum state to the GHZ-type entanglement of $\mathcal{S}+\mathcal{M}$ with environment $\mathcal{E}$. We investigated the process of disentangling the two-mode squeezed state and found that there are the two distinct routes of the disentangling process: two-way and GHZ entanglement. After a little algebra, we can easily prove that, at the instance of disentanglement, the mutual information between the two modes a_1 and a_2 is higher when it is disentangled through two-way entanglement with the environment than GHZ entanglement. This means that more information on a_1 is gained by the "measurement" of mode a_2 [28]. For the study of decoherence, we have shown that the infinite environmental variables are reducible to the considerably fewer effective-variables that give the equivalent description of the dynamics of the system. The reduction enables one to investigate and control the decoherence in an experiment, and further to analyze the nature of entanglement the system and the environment.

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