Study of atomic entanglement through the evolution of the field in cavity QED

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Abstract

We consider the cavity field interacting with two unidentical atoms in a perfect cavity. We study the evolution of the quasiprobability distribution for the field with a view to understand atomic entanglement. Due to the atom-field coupling the Gaussian peak of the initial coherent state evolves into one, two, or three peaks depending on the initial entanglement between two atoms. The dynamical evolution of the field also depends on the statistical nature of the state of the atoms.

32.80-t, 42.50.-p, 42.50.Fx

I. INTRODUCTION

The entangled states have been very important in the foundations of quantum mechanics and in the field of quantum optics. The entangled states of the radiation field as well as atoms have been important [1–5] in subjects like EPR paradox, Bell's inequalities, quantum teleportation, quantum computing and cryptography. In this work, we examine how the atoms in an entangled state interact with the radiation field in a cavity and how the field distribution itself is a reflection of the atomic entanglement. Note that the interaction of atoms with the field in the cavity has been extensively studied and several reviews on this subject exist [6]. In the context of cavity QED [7], the study of this interaction with atoms in entangled states has been studied [4,8,9]. The coherent interaction between two identical atoms has also been discussed [10,11]. In the present paper, we would especially focus on the new results that emerge, if the two atoms have different frequencies [12]. This would be the case for example, if two different species of isotopes are coupled to a single cavity mode.

The organisation of this paper is as follows - In Section II, we write the Hamiltonian and discuss the various entangled states. We derive an analytical expression for the evolution operator for the two atom system. The atoms can be in any of the four Bell states. In Section III, we present numerical results for the dynamical evolution of the field. The field is initially in a coherent state. The field evolution is most conveniently presented in terms of the quasiprobability distributions [13,14]. We present numerical results for the evolution of the Q-function of the field. The evolution of the Q-function is shown to be strongly *dependent on the entanglement* between two atoms.

II. TIME EVOLUTION OPERATOR-ANALYTICAL RESULTS

We consider a pair of unidentical two-level atoms A and B interacting with a single-mode cavity field which is quantized. The excited and ground states are, respectively, denoted by $|e_{A,B}\rangle_a$ and $|g_{A,B}\rangle_a$ for the atoms A and B. When atoms interact with a field, the whole system consists of two subsystems, viz, the atom and the field. Denoting the atomic subsystem by the subscript a and the field by f the density matrix ρ of the total system at time $t = 0$ is $\rho = \rho_a \otimes \rho_f$. In this paper we assume that the atom-field couplings

κ are equal for each atom and the atoms are oppositely detuned from the resonant cavity frequency, i.e., the atom-field detunings, Δ_A and Δ_B, are related by $\Delta_A = -\Delta_B \equiv \Delta$. Let a , $a^\dagger$ be the annihilation and creation operators for the cavity field. Under this assumption, the Hamiltonian of the system in the frame rotating with the cavity frequency is

$$H = \sqrt{2}\hbar\kappa(\hat{a}^\dagger\hat{\sigma}_s^- + \hat{a}\hat{\sigma}_s^+) - \hbar\Delta\hat{\sigma}^o \tag{1}$$

where the transition operator $\hat{\sigma}_s^+$ is defined as

$$\hat{\sigma}_s^+ = \frac{1}{\sqrt{2}}(|e_A\rangle_{a\,a}\langle g_A| + |e_B\rangle_{a\,a}\langle g_B|) \tag{2}$$

and $\hat{\sigma}_s^-$ is its hermitian conjugate. We have also introduced the new operator

$$\hat{\sigma}^o = |g_A\rangle_{a\,a}\langle g_A| - |g_B\rangle_{a\,a}\langle g_B|. \tag{3}$$

Both the atoms can be in their ground states $|g_A, g_B\rangle_a$ or in their excited states $|e_A, e_B\rangle_a$. When only one atom is excited the atom is either in the symmetric state $|s\rangle_a$ or in the antisymmetric state $|a\rangle_a$, defined as

$$|s\rangle_a = \frac{1}{\sqrt{2}}(|e_A\rangle_a|g_B\rangle_a + |g_A\rangle_a|e_B\rangle_a) \tag{4}$$

and

$$|a\rangle_a = \frac{1}{\sqrt{2}}(|e_A\rangle_a|g_B\rangle_a - ||g_A\rangle_a|e_B\rangle_a). \tag{5}$$

As a special case, it is easily seen that when the two identical atoms are resonantly coupled with the single-mode cavity field, i.e., $\Delta = 0$, the atomic evolution is restricted to the ground-ground, symmetric and excited-excited states. However, for unidentical atoms with $\Delta \neq 0$ we will see that the operator $\hat{\sigma}^o$ plays a crucial role and leads to qualitatively different behavior from that found in the case of identical atoms. The reason for this is that it generates a coherent coupling of the singly-excited symmetric and antisymmetric atomic states, as evident by

$$\hat{\sigma}^o|s\rangle_a = -|a\rangle_a \text{ and } \hat{\sigma}^o|a\rangle_a = -|s\rangle_a. \tag{6}$$

This, of course, is not a cavity-induced effect but merely represents the different evolution frequencies of the individual atoms modulating their relative phase. It must be noted that the interaction of two identical atoms with a cavity field is studied in [15] that of unidentical atoms in [12]. However previous works did not focus on the effects which result from the entanglement of two atoms.

The density operator for the combined atom-field system follows a unitary time evolution generated by the evolution operator,$\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$. The evolution operator has been extensively studied for a two-and a three-level atom Jaynes-Cummings model [6]. However, for a pair of unidentical two-level atoms, the four atomic levels are coupled with the field. For this complicated case we derive the evolution operator. With the use of the Hamiltonian (1), the evolution operator $\hat{U}(t)$ is written in the atomic basis

$$\hat{U}(t) = \sum_{n=0}^{\infty} \frac{(-it)^n}{n!} \begin{pmatrix} 0 & \sqrt{2}\kappa\hat{a} & 0 & 0 \\ \sqrt{2}\kappa\hat{a}^\dagger & 0 & \Delta & \sqrt{2}\kappa\hat{a} \\ 0 & \Delta & 0 & 0 \\ 0 & \sqrt{2}\kappa\hat{a}^\dagger & 0 & 0 \end{pmatrix}^n \tag{7}$$

where the atomic states are represented in the matrix form by

$$|e_A, e_B\rangle_a = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |s\rangle_a = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad |a\rangle_a = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad |g_A, g_B\rangle_a = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{8}$$

Using the commutation relations and Taylor expansions of sine and cosine functions we find the analytical form of the evolution operator:

$$\hat{U}(t) =$$

$$\begin{pmatrix} 2\kappa^2\hat{a}(\hat{C}-\hat{\Theta})\hat{a}^\dagger + 1 & -i\sqrt{2}\kappa\hat{a}\hat{S} & \sqrt{2}\kappa\Delta\hat{a}(\hat{C}-\hat{\Theta}) & 2\kappa^2\hat{a}(\hat{C}-\hat{\Theta})\hat{a} \\ -i\sqrt{2}\kappa\hat{S}\hat{a}^\dagger & \cos\hat{\Omega}t & -i\Delta\hat{S} & -i\sqrt{2}\kappa\hat{S}\hat{a} \\ \sqrt{2}\kappa\Delta(\hat{C}-\hat{\Theta})\hat{a}^\dagger & -i\Delta\hat{S} & \Delta^2\hat{C} + 2\kappa^2\hat{\Theta}(2\hat{n}+1) & \sqrt{2}\kappa\Delta(\hat{C}-\hat{\Theta})\hat{a} \\ 2\kappa^2\hat{a}^\dagger(\hat{C}-\hat{\Theta})\hat{a}^\dagger & -i\sqrt{2}\kappa\hat{a}^\dagger\hat{S} & \sqrt{2}\kappa\Delta\hat{a}^\dagger(\hat{C}-\hat{\Theta}) & 2\kappa^2\hat{a}^\dagger(\hat{C}-\hat{\Theta})\hat{a} + 1 \end{pmatrix} \tag{9}$$

where $\hat{n}$ is the photon number operator $\hat{a}^\dagger\hat{a}$ and the time-dependent operators $\hat{C}$ and $\hat{S}$ are defined by

$$\hat{C} = \hat{\Theta}\cos\hat{\Omega}t \quad \text{and} \quad \hat{S} = \hat{\Omega}^{-1}\sin\hat{\Omega}t, \tag{10}$$

where

$$\hat{\Omega}^2 = \hat{\Theta}^{-1} = 4\kappa^2\hat{n} + (\Delta^2 + 2\kappa^2). \tag{11}$$

The evolution of the reduced density operator for the field is given by tracing over the atomic variables:

$$\hat{\rho}_f(t) = \mathrm{Tr}_a[\hat{U}(t)\hat{\rho}(0)\hat{U}^\dagger(t)]. \tag{12}$$

Once the analytical form of its density operator is at hand, the statistical properties of the field may easily be studied.

III. SIGNATURES OF ATOMIC ENTANGLEMENT IN THE CAVITY FIELD DISTRIBUTION

In order to study the quantum statistical properties of the field it is instructive to examine the quasiprobably distribution for the field. These distributions have become especially important in recent times as one could use them to demonstrate interference effects which are of quantum origin [16]. For example, for a Schrödinger cat state the Wigner function, which is one out of the many possible quasiprobability distributions becomes negative and exhibits oscillations in certain regions of phase space. The quasiprobability distributions have not remained merely the theoretical tools but we now have many theoretical proposals and experimental measurements of the quasiprobability distributions [14]. In this paper we use extensively the Q-function. The Q function, $Q(\alpha, t)$, is a quasiprobability distribution defined as the coherent state expectation value of the field density matrix:

$$Q(\alpha, t) = \frac{1}{\pi} {}_f\langle\alpha|\hat{\rho}|\alpha\rangle_f. \tag{13}$$

where $|\alpha\rangle_f$ denotes a coherent state of the field [13]. We note that for the Jaynes-Cummings model [6], i.e., a single quantized electromagnetic field interacting with a two-level atom in a lossless cavity, Risken and Eiselt [17] have studied the Q function of the cavity field and investigated the relation between the evolution of this quasiprobability distribution and the collapses and revivals of Rabi oscillations in the atomic inversion. The dynamics of the Q function also manifests in the formation of a quantum superposition between two coherent states during the evolution of the Jaynes-Cummings field [18]. Clearly, it would be interesting to find the *signatures of cooperativity* in the evolution of the quasiprobability distributions like the Q-function.

Assuming that the atoms are initially in their symmetric state (4) and the initial field is described by the density operator $\hat{\rho}_f$, and using of Eq.(12) we obtain the time-dependence of the field

$$\hat{\rho}_f(t) = 2\kappa^2 \hat{a}\hat{S}\hat{\rho}_f\hat{S}^\dagger\hat{a}^\dagger + \cos\hat{\Omega}t\hat{\rho}_f\cos\hat{\Omega}t + \Delta^2\hat{S}\hat{\rho}_f\hat{S}^\dagger + 2\kappa^2\hat{a}^\dagger\hat{S}\hat{\rho}_f\hat{S}^\dagger\hat{a}\ . \tag{14}$$

This is to be used in (13) to study the time evolution of the quasiprobability distribution $Q(\alpha, t)$. The analytical formula is rather long and therefore we do not present it. In this paper, we are interested in the initial preparation of the cavity field in a coherent state with mean photon number $\bar{n}$ and the phase ϕ. This state is represented by the Poissonian-weighted summation of the Fock states $|n\rangle_f$:

$$|\beta\rangle_f = \mathrm{e}^{-\bar{n}/2}\sum_n \frac{(\sqrt{\bar{n}}\mathrm{e}^{i\phi})^n}{\sqrt{n!}}|n\rangle_f\ . \tag{15}$$

Using (15) in (14) calculations show that the mean photon number of the time-dependent cavity field is given by

$$\bar{n}(t) = \sum_n P(n)\Big\{2\kappa^2\frac{\sin^2\Omega_{n+1}t}{\Omega_{n+1}^2}(n+1) + 2\kappa^2\frac{\sin^2\Omega_{n-1}t}{\Omega_{n-1}^2}n + \cos^2\Omega_n t + \Delta^2\frac{\sin^2\Omega_n t}{\Omega_n^2}\Big\}, \tag{16}$$

where $P(n)$ is the Poissonian photon-number distribution of the initial coherent state. As for the single-atom Jaynes-Cummings model, the mean photon number shows collapses and revivals of the Rabi oscillations. If the mean photon number $\bar{n}$ of the initial coherent field is large, the Rabi frequency is

$$\Omega_{\bar{n}} = \sqrt{4\kappa^2\bar{n} + (\Delta^2 + 2\kappa^2)}, \tag{17}$$

and the revival time t_R is calculated as

$$t_R = \frac{\pi}{\Omega_{\bar{n}+1} - \Omega_{\bar{n}}} \approx \frac{\pi\sqrt{\bar{n} + \Delta^2/4\kappa^2}}{\kappa} \tag{18}$$

where the approximation applies when $\bar{n} \gg 1$. Under this assumption, the first and last terms in the right-hand side of Eq. (16) are nearly same in the relatively short interaction time as $\bar{n} \approx \bar{n} + 1 \gg 1$. When the field is resonant with both the atoms, i.e., $\Delta = 0$, we find that the evolution of the cavity field is similar to that for the single atom Jaynes-Cummings model. However, when the atoms are oppositely detuned, the evolution of the field is considerably different from the case of the non-resonant interaction of a single atom in the cavity. Further the evolution depends strongly on the initial atomic entanglement as we show in the following.

(a) Symmetric Excitation:

Using the density matrix (14) in the definition (13), we plot the dynamics of the Q function for a given detuning in Fig.1. It is well-known that the Q function of the initial coherent state is the Gaussian peak centered at the phase space point $\sqrt{\bar{n}}\exp(i\phi)$. The calculation shows that two equally-weighted peaks counter-rotate with respect to each other and collapse at the other side of the phase space. This result is independent of the detuning Δ. This can be compared with Fig.2 where we show the dynamics for one atom in the cavity (see [19], for example). In this case, as shown in Fig.2, the initial coherent state splits into two peaks whose weights are determined by the *sign and size of the detuning* between the field and the atom. For $\Delta = 10$ the peak rotating counter-clockwise is more pronounced than the other peak. On the contrary, if $\Delta = -10$, a bigger weight will be given to the peak rotating clockwise. For the two-atom problem one might consider the evolution as due to oppositely detuned atoms. These two evolutions interfere constructively to produce equally weighted peaks.

(b) Antisymmetric Excitation:

Similarly, using Eqs.(9),(12),and (13) we plot the Q function in Fig.3 at half of the revival time when the field is initially in the coherent state and the atoms are in the antisymmetric state (5). When the two identical atoms are resonant with the field, the initial Gaussian peak remains. For each atomic interaction, the initial Q function splits into two equally-weighted peaks but they destructively interfere leading to no evolution. This well-known result can also easily be seen from the Hamiltonian (1). The antisymmetric state is decoupled from the other atomic states for $\Delta = 0$, so that the initial cavity field remains unchanged during the whole evolution. When the two atoms are detuned, each atom causes unequally-weighted splitting of the peaks in the Q function. The weights depend on the sign of the detuning.

In this paper, the two atoms are not identical and they are oppositely-detuned so that each atom gives counter weights to the peaks. If the two atoms are initially prepared in the antisymmetric state, the rotating peaks destructively interfere which results in two small peaks counter-rotating each other in phase space while a peak with large weight still remains at the initial point. The situation is quite similar at other fractions of revival times like $t_R/3$ and $t_R/4$.

(c) Mixed State Excitation:

The symmetric and antisymmetric states (4,5) are, in fact, two of the all possible entangled states. We have seen that different initial entanglement of atomic states results in different evolutions of the field states. As a comparison, let us consider that the atoms are initially in an incoherent mixture whose density matrix is written as

$$\rho_a^m = \frac{1}{2}(|s\rangle_{a\,a}\langle s| + |a\rangle_{a\,a}\langle a|) = \frac{1}{2}(|e_A, g_B\rangle_{a\,a}\langle g_B, e_A| + |g_A, e_B\rangle_{a\,a}\langle e_B, g_A|). \tag{19}$$

Assuming that the field is initially in the coherent state, the Q function is plotted in Fig.4 at half of the revival time. We can see that the three peaks are pronounced due to the mixture. Even when the atoms resonantly interact with the field, there are three peaks in phase space at the half revival time. The counter rotating peaks are from the initial symmetric state and the remaining peak is due to the initial antisymmetric state. The height for the rotating peaks is a half of that for the remaining peak. When the atoms are detuned from the cavity field, the initial antisymmetric state also contributes to the rotating peaks so that the rotating peaks grow in magnitude as shown in the figure 4b.

(d) Excitation in other Bell States:

In addition to the symmetric and antisymmetric states (4,5), there are *two other* entangled (Bell) states, $|\Phi^{\pm}\rangle$ defined as

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|e_A, e_B\rangle \pm |g_A, g_B\rangle). \tag{20}$$

Note that the entangled states (20) involves the simultaneous excitation of both atoms, whereas in the states (4,5) only one atom was excited. When the atoms are in the entangled state $|\Phi^+\rangle$, the initial coherent cavity field evolves as in Fig.5, which shows that the initial Gaussian peak splits into two which counter-rotate each other like for the initial preparation of the atoms in the symmetric state (Fig.1). The atomic entangled state $|\Phi^-\rangle$ together with the field coherent state is nearly an eigenstate of the Hamiltonian (1) so that when the atoms are initially prepared in the entangled state $|\Phi^-\rangle$ we expect no dynamical evolution of the Q function.

(e) Uncorrelated Atoms:

The dynamical evolution of the Q-function for *both atoms initially* in the excited state is shown in the Fig.6 for $t = t_R/2$ and t_R. There is no full-scale revival at t_R. The revival in fact occurs at $t = 2t_R$.

IV. CONCLUSIONS

We have studied the dynamics of a system of two atoms (which are not necessarily identical) in a single mode cavity with a view to uncover the entanglement between atoms. We obtained analytical solution to the evolution operator for the coupled atomic-field system. This generalizes the known solution of Jaynes-Cummings model to an effectively four level detuned system. We studied the dynamics of the quasiprobability distributions in order to visualize the atom-field interaction very clearly. Our numerical results reveal the effects of the initial *entanglement* between the two atoms. For example, compare Figs (1), (3) and (4b) for symmetric, antisymmetric and mixed excitations of the atomic systems.

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FIGURES

FIG. 1. Contour plots of the Q functions at the initial time, at $t_R/4$, $t_R/2$, and t_R where t_R is the revival time. The atoms are initially in the symmetric state and the field is in the coherent state with mean photon number $\bar{n} = 49$ and with $\phi = 0$. The detuning $\Delta = 10$. All frequencies are in units of the coupling constant κ.

FIG. 2. Contour plots of the Q functions at the atom-field interaction times at $t_R/2$ and t_R. The field is initially prepared as in Fig.1 to interact with a single atom in the cavity with the detuning $\Delta = 10$. The atom is initially in the excited state.

FIG. 3. The Q function at half of the revival time $t_R/2$ for the cavity field, for atoms initially in the antisymmetric state and $\bar{n} = 49, \Delta = 10$.

FIG. 4. The Q function at half of the revival time $t_R/2$ for the cavity field for atoms initially in the mixed state $\hat{\rho}_a^m = \frac{1}{2}(|e_A, g_B\rangle \langle g_B, e_A| + |g_A, e_B\rangle \langle e_B, g_A|)$ and the field in the coherent state with $\bar{n} = 49$. The detunings are $\Delta = 0$ (a) and $\Delta = 10$ (b).

FIG. 5. The Q function at half of the revival time $t_R/2$ for the cavity field for atoms initially in the Bell state $|\Phi^+\rangle$ and $\bar{n} = 49, \Delta = 0$.

FIG. 6. The Q function for the cavity field at half of the revival time $t_R/2$ (a) and at the revival time t_R (b) for atoms initially in their excited states and $\bar{n} = 49, \Delta = 10$.