Homotopy-theoretic models for toric varieties

supervised by

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The aim of the project is to investigate stable homotopy models for toric varieties. More precisely, the aim is to describe the so-called derived category of a toric variety by means of a construction similar to that of "spectra" in stable homotopy theory.

Toric varieties are a class of geometric objects allowing hands-on, combinatorial descriptions in terms of "fans", that is, partitions of $\mathbb{R}^n$ into convex cones; projective spaces are the most prominent examples. Toric varieties are much-studied objects in algebraic geometry, see for example [Oda88, Ful93].

In the case of regular toric varieties the derived category admits a combinatorial description of sorts [Hüt10]. More excitingly, it has been shown recently in a PhD thesis by Corvan that the derived category of projective spaces can be modelled in stable homotopy theory using an elaborate notion of "multi-spectra". A spectrum is a kind of generalised topological space, encoded by a sequence of spaces $(X_n)_{n\geq 0}$ and maps $\Sigma X_n \to X_{n+1}$ from the suspension of X_n to the next space X_{n+1}. A multi-spectrum is again a sequence of spaces, but equipped with several maps $\Sigma X_n \to X_{n+1}$ which interact in a suitable way.

The aim of the proposed project is to refine and extend Corvan's construction, with a view towards capturing more of the structure of the derived categories on the one hand, and modelling derived categories of the much larger class of regular toric varieties on the other hand.

At the start of the project, the student will need to learn the language of model categories, an important framework for abstract stable homotopy theory and homological algebra, and become familiar with the basic terms of algebraic geometry. Depending on preference and preliminary investigations, the project may then concentrate initially on more algebraic considerations (*e.g.*, modelling "twisted" versions of projective spaces and more general toric varieties), or on more topological aspects (*e.g.*, the use of different types of spectra to model derived categories). These strands are not independent, and will be joined eventually to provide a satisfying stable-homotopy model for derived categories of toric varieties.

References

[Ful93] William Fulton. *Introduction to toric varieties*, volume 131 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 1993. The William H. Roever Lectures in Geometry.

[Hüt10] Thomas Hüttemann. On the derived category of a regular toric scheme. *Geom. Dedicata*, 148:175–203, 2010.

[Oda88] Tadao Oda. *Convex bodies and algebraic geometry*, volume 15 of *Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)]*. Springer-Verlag, Berlin, 1988. An introduction to the theory of toric varieties, Translated from the Japanese.