

Comparing the orthogonal and embedding calculi

supervised by

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This project considers an interaction between the homotopy theory of topological spaces and embeddings of manifolds. The starting point is collection of spaces $T(\mathbb{R}^n, M)$, one for each $n \geq 0$ and for each manifold M in some fixed collection of interest. For example, we could have $T(\mathbb{R}^n, M) = \mathbb{R}^n \times M$ or $T(\mathbb{R}^n, M) = \text{map}(M, \mathbb{R}^n)$.

If we fix M and allow n to vary, then the standard way to study these collection of spaces is a method known as the orthogonal homotopy calculus [Wei95]. On the other hand if we fix n and allow M to vary, the usual method to study this collection of spaces is known as the embedding calculus, [Wei96].

Recent work has formalised these two tools into the modern language of model categories, [BO13], [BdBW13]. This project is concerned with combining these two methods and studying the interaction of the model categories. Once the formal structure has been made clear, the next task will be to look at how several existing calculations [ALV07] fit into the new structure, with an eye to extending them using our new understanding.

The student should have attended courses on algebraic topology and topology. Some algebra will also be useful. This project will require the student to become familiar with the abstract language of model categories [DS95] and modern categories of spectra [MM02].

References

- [ALV07] G. Arone, P. Lambrechts, and I. Volić. Calculus of functors, operad formality, and rational homology of embedding spaces. *Acta Math.*, 199(2):153–198, 2007.
- [BdBW13] P. Boavida de Brito and M. Weiss. Manifold calculus and homotopy sheaves. *Homology Homotopy Appl.*, 15(2):361–383, 2013.
- [BO13] D. Barnes and P. Oman. Model categories for orthogonal calculus. *Algebr. Geom. Topol.*, 13(2):959–999, 2013.
- [DS95] W. G. Dwyer and J. Spaliński. Homotopy theories and model categories. In *Handbook of algebraic topology*, pages 73–126. North-Holland, Amsterdam, 1995.
- [MM02] M. A. Mandell and J. P. May. Equivariant orthogonal spectra and S -modules. *Mem. Amer. Math. Soc.*, 159(755):x+108, 2002.
- [Wei95] M. Weiss. Orthogonal calculus. *Trans. Amer. Math. Soc.*, 347(10):3743–3796, 1995.
- [Wei96] M. Weiss. Calculus of embeddings. *Bull. Amer. Math. Soc. (N.S.)*, 33(2):177–187, 1996.