Rational G-equivariant homotopy theory for profinite G.

supervised by

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Topological spaces occur almost everywhere in mathematics. It has been particularly fruitful to study homotopies of these spaces (and maps between them). Symmetries appear everywhere in mathematics, so it is logical to study topological spaces that have symmetries. In turn we should look at homotopies that preserve these symmetries. This area is known as equivariant homotopy theory.

Equivariant homotopy theory is particularly exciting when one considers groups of symmetries that have their own topology. To make the theory tractable, one usually assumes that the groups are compact. There are two large classes of compact groups that have been useful: the compact Lie groups, such as $O(n)$ or $U(n)$, and the profinite groups, like the p-adic integers.

Much attention has been paid to the case of compact Lie groups and there are many interesting results in this setting. The case of profinite groups has received less attention. This project intends to remedy this, by continuing work of Fausk [Fau08] and the adviser [Bar11]. The aim of this project would to be establish the standard results of equivariant stable homotopy theory in the profinite case. For example, the student would investigate to what extent the the Adams isomorphism and Wirthmueller isomorphism hold in the profinite case, or would consider the multiplicative properties of such spectra.

The student should have attended courses on algebraic topology and topology. Some algebra will also be useful. This project will require the student to become familiar with the abstract language of model categories [DS95] and modern categories of spectra [MM02] and [MMSS01].

References

[Bar11] D. Barnes. Rational $\mathbb{Z}_p$–equivariant spectra. *Algebr. Geom. Topol.*, 11(4):2107–2135, 2011.

[DS95] W. G. Dwyer and J. Spaliński. Homotopy theories and model categories. In *Handbook of algebraic topology*, pages 73–126. North-Holland, Amsterdam, 1995.

[Fau08] Halvard Fausk. Equivariant homotopy theory for pro-spectra. *Geom. Topol.*, 12(1):103–176, 2008.

[MM02] M. A. Mandell and J. P. May. Equivariant orthogonal spectra and S-modules. *Mem. Amer. Math. Soc.*, 159(755):x+108, 2002.

[MMSS01] M. A. Mandell, J. P. May, S. Schwede, and B. Shipley. Model categories of diagram spectra. *Proc. London Math. Soc. (3)*, 82(2):441–512, 2001.