Positive extensions

supervised by

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A *positive definite function* defined on the real line is a complex-valued function $f : \mathbb{R} \to \mathbb{C}$ such that for any real numbers $x_1, \ldots, x_n$ the $n \times n$ matrix $A = (a_{ij})_{i,j=1}^n$, where $a_{ij} = f(x_i - x_j)$, is positive semi-definite. Positive definiteness of a function arises naturally in the theory of Fourier transforms; it is easy to see that to be positive definite it is sufficient for the function f to be the Fourier transform of a positive function defined on the real line.

In general one can define positive definite functions (or called positive type functions) on any locally compact topological group and on any groupoid. Positive type functions on groups (on groupoids, respectively) occur naturally in the representation theory of groups on Hilbert spaces (of groupoids on Banach spaces, respectively). There are interesting commutative Banach algebras arising from positive type functions as well.

The aim of the project is to study some extension problems for partially defined positive definite functions on the above structures. As a first step, we will investigate the notion of positive type functions being "partially defined". While there are partial results in the setting of groups, these problems are wide open, especially in the setting of groupoids. They have connections with the completion problems for operators and matrices that will be explored after the core results are obtained.

References

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