Approximation of intrinsic volumes in stereology

supervised by

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Imagine you make an observation about the shape of certain cells in a human body or about plants in your greenhouse. Feeling very excited, you take a digital picture with your camera or microscope to further investigate this observation on the computer. If your shapes are sufficiently well behaved and belong to a space of shapes $\mathbb{K}^n$, then it can be shown that there are exactly $n+1$ independent functionals $\phi : \mathbb{K}^n \to \mathbb{R}$ that are *additive*, *motion invariant* and *continuous* on $\mathbb{K}^n$. This result is known as *Characterisation Theorem of Hadwiger*. All of these functionals include and behave like the ordinary volume of a body and are therefore called its *intrinsic volumes*. For example, the intrinsic volumes of a three-dimensional object are the volume, the surface area, the integrated mean curvature and the Euler characteristic.

But how to quantify intrinsic volumes of a body? How to take reliable geometric measurements from digital images?

It turns out that this is a highly non-trivial mathematical question and leads into the field of stochastic geometry and stereology. Stereology is the science of inferring geometric information, such as intrinsic volumes, of a three-dimensional object from sections with two-dimensional planes or one-dimensional lines.

The aim of this project is to first survey, implement and compare different algorithms used in stereology. This first step will set the ground for further investigations. Depending on the individual interests this project can then proceed into an applied direction, i.e. a collaboration with biologists or pathologists. Alternatively, there are various theoretical questions of interest: How to reduce the variance of the involved estimators by choosing smarter sampling schemes? How to reconstruct a three-dimensional object in an efficient way? How to relate real-world objects in a reliable way to reconstructions? How to improve the complexity of the algorithms?

A student that is interested in this project should have a profound knowledge of linear algebra and good programming skills. Moreover, in-depth knowledge of discrete mathematics and stochastics will be helpful. Work on this project will require to study convex and integral geometry as well as combinatorial and computational topology.

References

[1] H. Edelsbrunner, F. Pausinger, *Stable Length Estimates of Tube-Like Shapes*, J Math Imaging Vis 50 (2014), 164–177.

[2] J. González-Villa, M. Cruz, L.M. Cruz-Orive, *On the precision of the nucleator*, Image Anal Stereol 36 (2017), 121–132.

[3] Ó. Thórisdóttir, A. H. Rafati, M. Kiderlen, *Estimating the surface area of nonconvex particles from central planar sections*, J Microscopy 255 (2014), 49–64.