An infinite dimensional Sandwich Theorem

supervised by

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A *graph* is a set of points called vertices, and a set of edges, each edge connecting a pair of given vertices. Graphs have many applications in Computer Science and Engineering, and are studied from a number of perspectives. They are also fundamental in the area of Zero-error Quantum Information Theory, where they are used to encode properties of communication channels. Two fundamental parameters of a graph G are its chromatic number $\chi(G)$ (the minimum number of colours that can be assigned to vertices in such a way that adjacent vertices receive different colours) and its clique number $\omega(G)$ (the maximum number of vertices each pair of which is an edge). These parameters have high computational complexity, and therefore the discovery by L. Lovász of a parameter, known today as the Lovász number of the graph and denoted by $\theta(G)$, that lies in between and can be computed in polynomial time, brought about spectacular progress and continues up to date to inspire further investigations. The inequalities

$$\omega(G) \leq \theta(G) \leq \chi(\bar{G})$$

(where $\bar{G}$ is the complement of G) are known as the *Sandwich Theorem* [1].

The aim of this project is to investigate infinite dimensional versions of the Sandwich Theorem. The specific setting in which the Theorem will be formulated and proved will be made precise during the course of work, as it requires additional knowledge in Operator Theory. As consequences, we will obtain applications in Zero-error Information Theory; indeed, our approach will allow us to study capacities of information channels as one-shot parameters of respective infinite dimensional objects that arise in the area of Operator Algebras.

The project lies on the border of Graph Theory, Functional Analysis and Zero-error Quantum Information Theory and will equip the prospective student with knowledge suitable for both theoretical investigations in Operator Algebra Theory, and pursuit of applications to Quantum Information Theory. To start working on the project, the student will need a sound background in Linear Algebra and Functional Analysis and be interested in Graph Theory.

References

[1] D. E. Knuth, *The sandwich theorem*, Electron. J. Combin. 1 (1994), A1.