Asymptotic variance of Newton-Cotes quadratures based on randomized sampling points

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Motivated by the stereological problem of estimating the volume of an object from its sections with parallel planes, we discuss classical Newton-Cotes quadrature rules, where the sampling points are random. More precisely, if f is an integrable function with bounded support on the real line, we consider a symmetrized n-th order Newton-Cotes approximation $\hat{V}_n$ of $V = \int f \mathrm{d}x$ based on the values $(f(x_i))_{i \in \mathbb{Z}}$, where $X = \{x_i | i \in \mathbb{Z}\}$ forms a stationary point process on $\mathbb{R}$.

As the sampling is random, $\hat{V}_n$ is a random variable. Under mild assumptions, we show that $\hat{V}_n$ is unbiased for V and determine the asymptotic variance behavior of $\hat{V}_n$ as the sampling is refined. This behavior depends on the choice of X and on the smoothness of f. Interestingly, refined Euler-MacLaurin formulae that were used in related work for the equidistant sampling scheme, were not sufficient for the proofs. We therefore adapted the classical Peano kernel representations to our setting.

The talk will be concluded with an application of the findings to the stereological problem mentioned above, yielding a family of estimators that generalize the classical Cavalieri procedure.

Joint work with Mads Stehr, CSGB, Aarhus University.

Sampling and applications

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Sampling may be defined as the action of observing the value of one or more variables following a fixed scheme. We can distinguish two different approaches:

- Random sampling, in which samples are taken randomly in a certain region.
- Systematic sampling, where samples have some structure, an example may be a geometrical probe over $\mathbb{R}^3$.

Each intended application carries its own challenges; fast convergence, few representative samples, probabilistic estimation of the error etc.

In the first part of the talk, we will introduce a new approximation to the classic nucleator estimator, which is a systematic sampling approach to approximate the volume of a random particle in $\mathbb{R}^3$.

The second part is devoted to True Random Number Generators (TRNGs) that exploit physical processes as a source of randomness as well as the commercial interest in computer models that simulate true random number generators.

Finally, we will discuss an approach that aims to combine the good properties associated to systematic sampling with the ones associated random sampling.