Sheaf models for equivariant stable homotopy theory

supervised by

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Topological spaces occur almost everywhere in mathematics. It has been particularly fruitful to study homotopies of these spaces (and maps between them). Symmetries appear everywhere in mathematics, so it is logical to study topological spaces that have symmetries. In turn we should look at homotopies that preserve these symmetries. This area is known as equivariant homotopy theory.

Equivariant homotopy theory is particularly exciting when one considers groups of symmetries that have their own topology. To make the theory tractable, one usually assumes that the groups are compact. There are two large classes of compact groups that have been useful: the compact Lie groups, such as $O(n)$ or $U(n)$, and the profinite groups, like the p-adic integers.

Recent work of the adviser and a PhD student [BS20a], [BS20b], has constructed a link between (stable) equivariant homotopy and the theory of sheaves with a group action in the case where G is a profinite group. Many questions on this topic remain, particularly those connected to monoidal structures (a form of categorical multiplication, inspired by the Cartesian product of topological spaces), change of groups functors and to what extents these results can be applied to compact groups in general.

The student should have attended courses on algebraic topology and topology. Some algebra will also be useful. This project will require the student to become familiar with the abstract language of model categories [DS95] and modern categories of spectra [MM02] and [MMSS01].

References

[BS20a] D. Barnes and D. Sugrue. The equivalence between rational G-sheaves and rational G-mackey functors for profinite G. Preprint, available as `arXiv:2002.11745`, 2020.

[BS20b] D. Barnes and D. Sugrue. Equivariant sheaves for profinite groups. Preprint, available as `arXiv:2012.03982`, 2020.

[DS95] W. G. Dwyer and J. Spaliński. Homotopy theories and model categories. In *Handbook of algebraic topology*, pages 73–126. North-Holland, Amsterdam, 1995.

[MM02] M. A. Mandell and J. P. May. Equivariant orthogonal spectra and S-modules. *Mem. Amer. Math. Soc.*, 159(755):x+108, 2002.

[MMSS01] M. A. Mandell, J. P. May, S. Schwede, and B. Shipley. Model categories of diagram spectra. *Proc. London Math. Soc. (3)*, 82(2):441–512, 2001.