Delayed feedback mechanisms

supervised by

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Many real world processes feed parts of their own outputs back to themselves as inputs. For instance, in the climate system there are mechanisms that can either amplify or diminish the effects of the initial drivers of climate. Since interactions between subsystems are often not instantaneous, predictive and faithful mathematical models of systems with feedback should incorporate past dependence. Indeed, delay differential equations are deterministic mathematical models of many history driven problems of practical importance [1, 2].

The phase space of delay differential equations is the Banach space of continuous functions $\phi : [-1, 0] \to \mathbb{R}$ with the norm given by $\|\phi\| = \max_{-1 \leq t \leq 0} |\phi(t)|$. That is, delay differential equations can be viewed as infinite dimensional dynamical systems. Although there is a wide variety of analytical, geometric and computational techniques to study the dynamics delay differential equations, there are many challenging open questions. For instance, the existence of the numerically observed chaos in

$$x'(t) = -\mu x(t) + \frac{px(t-\tau)}{1 + x(t-\tau)^n}, \qquad x \in \mathbb{R},\ \mu, p, n, \tau > 0,$$

the so-called Mackey-Glass equation [3], has not been proved yet.

The aim of the project is to study the dynamics of nonlinear differential equations with different types of time lags such as multiple, time-dependent and state-dependent. The student should have attended courses on Linear Algebra and Complex Variables, Metric and Normed Spaces, and Functional Analysis.

References

[1] Smith, H. L, *An introduction to delay differential equations with applications to the life sciences*, Springer New York, 2011.

[2] Erneux, T. *Applied delay differential equations* Springer Science & Business Media, 2009.

[3] Mackey, M. C. and Glass, L., *Oscillation and chaos in physiological control systems.*, Science, 197(4300):287-289, 1977.